

Logic 2: Modal Logic

Lecture 20

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Constant and variable domains

Constant and variable domains

If we combine the **tree rules** for K with the rules for classical propositional logic, we can prove the Barcan Formula and its converse.

The Barcan Formula and its converse are valid only if the domain of individuals is constant across accessible worlds.

The combined proof method is not complete with respect to constant-domain semantics: we can't prove $a \neq b \rightarrow \Box(a \neq b)$.

Constant and variable domains

If we combine the **axiomatic method** for K with the method for classical propositional logic, we can prove the Converse Barcan Formula but not the Barcan Formula.

We can't prove $a \neq b \rightarrow \Box(a \neq b)$.

Names in epistemic logic

Leibniz' Law:

A

$b = c$

$A[c//b]$

Names in epistemic logic

There seem to be clear counterexamples:

Hammurabi knows that Hesperus is visible in the evening sky.

Hesperus = Phosphorus.

Hammurabi knows that Phosphorus is visible in the evening sky.

$\Box Vh$

$h = p$

$\Box Vp$

Russell's (1905) response:

“...proper names are usually really descriptions. That is to say, the thought in the mind of a person using a proper name correctly can only be expressed explicitly if we replace the proper name by a description.”

Russell's (1905) response:

Hammurabi knows that Hesperus is visible in the evening sky.

Hesperus = Phosphorus.

Hammurabi knows that Phosphorus is visible in the evening sky.

$\Box \exists x (Hx \wedge \forall y (Hy \rightarrow x=y) \wedge \forall x)$

$h = p$ or $\exists x (Hx \wedge \forall y (Hy \rightarrow x=y) \wedge \exists y (Py \wedge \forall z (Pz \rightarrow z=y) \wedge x=y))$

$\neg \Box \exists x (Px \wedge \forall y (Py \rightarrow x=y) \wedge \forall x)$

The bullet-biting response:

Hammurabi really did know that Phosphorus is visible in the evening sky.

Follow-up problem:

- Hammurabi believed that Phosphorus is not visible in the evening sky.
- On the bullet-biting account, Hammurabi had inconsistent beliefs.
- We can't use Kripke semantics to model inconsistent beliefs.

Frege's response:

An individual can play many roles.

Every name is associated (not just with an individual but) with a role.

At every world, the name picks out whatever plays the associated role.

The role associated with 'Hesperus' is being the brightest body in the evening sky.

The role associated with 'Phosphorus' is being the brightest body in the evening sky.

At our world, Venus plays both of these roles.

At other worlds, different things play the two roles.

Frege's response:

Leibniz' Law is invalid.

$\Box Vh$

$h = p$

$\Box Vp$

Individual Concept Semantics

Individual Concept Semantics

We have assumed that the only function of a name or variable is to pick out an individual.

$V(a) = \text{Alice}$.

This renders Leibniz' Law valid.

A

$b = c$

$A[c//b]$

We now assume that the function of a name is to pick out a role.

- being the brightest body in the morning sky
- being the brightest body in the evening sky
- being the inventor of the zip
- ...

Such a role can be represented by a function from worlds to individuals.

Functions from worlds to individuals are called **individual concepts** or **intensional objects**.

Individual Concept Semantics

In **individual concept semantics**, the interpretation function assigns to every name an individual concept.

$V(a)$ = the function that maps every world w to the brightest object in the evening sky at w
= λw [the brightest object in the evening sky at w]

$M, w \models Fa$ iff $V(a)(w) \in V(F)$.

Leibniz' Law is no longer valid for modal sentences.

$\Box Vh$

$h = p$

$\Box Vp$

Individual Concept Semantics

In **individual concept semantics**, the interpretation function assigns to every name an individual concept.

$V(a)$ = the function that maps every world w to the brightest object in the evening sky at w
= λw [the brightest object in the evening sky at w]

$M, w \models Fa$ iff $V(a)(w) \in V(F)$.

The “necessity of identity” and the “necessity of distinctness” are also invalid.

(NI) $a = b \rightarrow \Box(a = b)$

(ND) $a \neq b \rightarrow \Box(a \neq b)$.

Problems:

1. $\Box\exists xA \rightarrow \exists x\Box A$ becomes valid.
2. There are no complete proof procedures.
3. Is every name associated with a unique role?
4. Might not help with other puzzle cases.

“Mary Ann Evans is George Elliot, but Smith doesn't know that she is.”

More puzzle cases

'Raiātea and Tahaa are different islands, but they might have been a single island.'

$$Hr \wedge Ht \wedge r \neq t \wedge \Diamond \exists x (Hx \wedge x = r \wedge x = t)$$

This entails:

$$r \neq t \wedge \Diamond (r = t)$$

Here, the “necessity of distinctness” appears to fail in circumstantial/metaphysical modality.

More puzzle cases

There are two tickets, numbered 1 and 2; one is blue, one is red; we don't know which colour goes with which number. We know that the blue ticket won.

1. Ticket 1 might have won. ($\Diamond Wt_1$)
2. Ticket 2 might have won. ($\Diamond Wt_2$)
3. These are all the tickets. ($\forall x(Tx \rightarrow (x=t_1 \vee x=t_2))$)
4. So: Any ticket might have won. ($\forall x(Tx \rightarrow \Diamond Wx)$)
5. The red ticket is a ticket. (Tr)
6. So: The red ticket might have won. ($\Diamond Wr$)

More puzzle cases

'If I were you I wouldn't accept the offer.'

$$a = b \quad \square \rightarrow Aa$$

More puzzle cases

Alice the time travel is about to travel back in time to meet her younger self.

As she arrives, is she young or old?

$P Fa?$

$P \neg Fa?$

$P(Fa \wedge \neg Fa)?$