Logic 2: Modal Logic

Lecture 16

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14 March 2023

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Review

Review

Indicative:

- If Oswald did not kill Kennedy, someone else did.

Subjunctive/counterfactual:

- If Oswald had not killed Kennedy, someone else would have.

Review

Two hypotheses about indicative conditionals

1. ‘if A then B’ means $A \rightarrow B$
2. ‘if A then B’ means $\Box(A \rightarrow B)A \strictif B$

Review

		$A \rightarrow B$	$A \strictif B$
Modus Ponens	if A then B, A $\therefore$ B	valid	valid
Conditional Proof	A entails B $\therefore$ if A then B	valid	valid
Or-to-If	$A \vee B$ $\therefore$ if not A then B	valid	invalid
Import-Export	if A then if B then C $\therefore\therefore$ if A and B then C	valid	invalid
Contraposition	if A then B $\therefore$ if not B then not A	valid	valid
Transitivity	if A then B, if B then C $\therefore$ if A then C	valid	valid
SDA	if A or B then C $\therefore$ if A then C and if B then C	valid	valid
Antec. Strength.	if A then C $\therefore$ if A and B then C	valid	valid
False Antec.	not A $\therefore$ if A then B	valid	invalid
True Cons.	B $\therefore$ if A then B	valid	invalid

Similarity semantics

Similarity semantics

- If Oswald had not killed Kennedy then someone else would have.

Intuitively, to assess a subjunctive conditional, we

1. rewind the world to the time of the antecedent,
2. make minimal changes to render the antecedent true,
3. then let history run its course.

The conditional is true iff the consequent is true at all the resulting worlds.

Similarity semantics

Different antecedents call for different revisions to the actual world.

- If Oswald had not killed Kennedy ...
- If Marilyn Monroe had killed Kennedy ...
- If Kennedy had died as an infant ...

$\Box(A \rightarrow C)$ entails $\Box((A \wedge B) \rightarrow C)$.

But

- If Oswald had not killed Kennedy then Kennedy would have been re-elected.

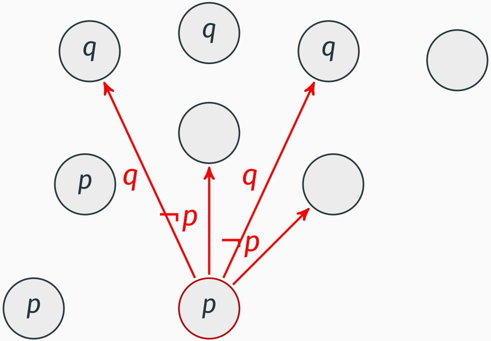
does not entail

- If Marilyn Monroe had killed Kennedy then Kennedy would have been re-elected.

Similarity semantics

p: Oswald kills Kennedy

q: Monroe kills Kennedy



Similarity semantics

Similarity semantics

$A \mathrel{\Box\!\!\rightarrow} B$ is true at w iff B is true at all the most similar A-worlds to w.

Similarity semantics

A **similarity model** consists of

- a non-empty set W of worlds,
- for each world w in W a similarity order $\prec_w$, and
- a function V that assigns to each sentence letter a subset of W.

Similarity semantics for $\Box\!\!\rightarrow$

If M is a similarity model and w a world in M, then

$M, w \models A \Box\!\!\rightarrow B$ iff $M, v \models B$ for all v such that (i) $M, v \models A$ and (ii) there is no $u \prec_w v$ with $M, u \models A$.

Similarity semantics

		$A \rightarrow B$	$A \mathrel{-\!\!\!3} B$	$A \mathrel{\Box\!\!\rightarrow} B$
Modus Ponens	if A then B, A ∴ B	valid	valid	valid
Conditional Proof	A entails B ∴ if A then B	valid	valid	valid
Or-to-If	$A \vee B$ ∴ if not A then B	valid	invalid	invalid
Import-Export	if A then if B then C ∴∵ if A and B then C	valid	invalid	invalid
Contraposition	if A then B ∴ if not B then not A	valid	valid	invalid
Transitivity	if A then B, if B then C ∴ if A then C	valid	valid	invalid
SDA	if A or B then C ∴ if A then C and if B then C	valid	valid	invalid
Antec. Strength.	if A then C ∴ if A and B then C	valid	valid	invalid
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True Cons.	B ∴ if A then B	valid	invalid	invalid

If-clauses as restrictors

If-clauses as restrictors

(1) If the murderer escaped through the window, there must be traces on the ground.

(2) If the murderer escaped through the window, there might be traces on the ground.

(1) should not be translated as $p \rightarrow \Box q$ or $p \strictif \Box q$. But $\Box(p \rightarrow q)$ works.

(2) cannot be translated as $\Diamond(p \rightarrow q)$. Better: $p \rightarrow \Diamond q$. Even better: $\Diamond(p \wedge q)$.

If-clauses as restrictors

(1) If it rains, we always stay inside.
(2) If it rains, we sometimes stay inside.
(3) If it rains, we usually stay inside.

(1) can't be translated as $p \rightarrow \Box q$ or $p \strictif \Box q$. But $\Box(p \rightarrow q)$ works.

(2) can't be translated as $p \rightarrow \Diamond q$ or $\Diamond(p \rightarrow q)$. But $\Diamond(p \wedge q)$ works.

(3) can't be translated as $p \rightarrow \mathrm{M}q$ or $\mathrm{M}(p \rightarrow q)$ or $\mathrm{M}(p \wedge q)$ or ....

If-clauses as restrictors

(1) If it rains, we always stay inside.
(2) If it rains, we sometimes stay inside.
(3) If it rains, we usually stay inside.

(1) says that in all situations in which it rains, we stay inside.

(2) says that in some situations in which it rains, we stay inside.

(3) says that in most situations in which it rains, we stay inside.

If-clauses as restrictors

(1) If the murderer escaped through the window, there must be traces on the ground.
(2) If the murderer escaped through the window, there might be traces on the ground.

(1) says that in all epistemically accessible worlds at which the murderer escaped through the window, there are traces on the ground.

(2) says that in some epistemically accessible worlds at which the murderer escaped through the window, there are traces on the ground.

If-clauses as restrictors

(1) Jones should help his neighbours.

(2) If Jones won't help his neighbours, he shouldn't tell them that he is coming.

(1) says that in the best of the circumstantially accessible worlds, Jones helps his neighbours.

(2) says that in the best of the circumstantially accessible worlds at which Jones won't help his neighbours, Jones doesn't tell them that he is coming.

If-clauses as restrictors

"The history of the conditional is the story of a syntactic mistake. There is no two-place *if...then* connective in the logical forms of natural languages. *If*-clauses are devices for restricting the domains of various operators. Whenever there is no explicit operator, we have to posit one."

— Angelika Kratzer, 1991

If-clauses as restrictors

(1) If Oswald didn't kill Kennedy, then someone else killed Kennedy.
(1b) If Oswald didn't kill Kennedy, then someone else must have killed Kennedy.

(1b) says that in all epistemically accessible worlds at which Oswald didn't kill Kennedy, someone else killed Kennedy.

This is equivalent to $p \strictif q$, with an epistemic accessibility relation.

If-clauses as restrictors

(2) If Oswald hadn't killed Kennedy, then someone else would have killed Kennedy.

Perhaps 'would' is a modal operator, meaning something like 'it is settled that'.

- She wrote a book. It would later become a bestseller.

Suppose 'would *q*' is true iff the laws of nature together with the current facts entail *q*.

So 'would *q*' is true at *w* iff *q* is true at all the closest worlds to *w*.

'If *p* would *q*' is true at *w* iff *q* is true at all the closest *p*-worlds to *w*.

This is equivalent to $p \mathrel{\Box\!\!\rightarrow} q$.