# Logic 2: Modal Logic

Lecture 15

Wolfgang Schwarz March 13, 2023

University of Edinburgh

# Conditionals in natural language

## Conditionals in natural language

- If Russia invades Estonia, NATO will attack Russia.
- If we don't reduce greenhouse gases, the climate might get out of control.
- You will be faster if you take a taxi.
- If Heisenberg hadn't undermined the Nazi's nuclear weapons programme, Germany would have won the war.
- If Jones hadn't untied the rope, Smith would not have fallen.

### Indicative:

• If Oswald did not kill Kennedy, someone else did.

### Subjunctive/counterfactual:

• If Oswald had not killed Kennedy, someone else would have.

# **Material conditionals**

 $A \rightarrow B$  is a "material conditional": it is true iff A is false or B is true.

Α	В	$A \rightarrow B$
1	1	1
1	0	0
0	1	1
0	0	1

The material analysis: 'If A then B' is a material conditional.

Α	В	if A then B
1	1	1
1	0	0
0	1	1
0	0	1

#### A quick argument for the material analysis

- 1. 'If A then B' entails 'not A or B'.
- 2. 'A or B' entails 'if not A then B'.
- 3. 'Not A or B' entails 'if not not A then B'.
- 4. 'Not A or B' entails 'if A then B'.

### Another quick argument for the material analysis

- 'If *n* is a prime number greater than 2 then *n* is odd.'
- 'For any number *n*, if *n* is a prime number greater than 2 then *n* is odd.'
- 'If 1 is a prime number greater than 2 then 1 is odd.'
- 'If 2 is a prime number greater than 2 then 2 is odd.'
- 'If 3 is a prime number greater than 2 then 3 is odd.'

• ...

#### Another argument, due to Alan Gibbard (1981)

Modus Ponens: 'If A then B' and A entail B.

Import-Export: 'If A then if B then C' is equivalent to 'if A and B then C'.

(1) 'If not A or B then if A then B.'

By Import-Export, (1) is equivalent to the tautology

(2) 'If [not A or B] and A then B.'

So (1) is a logical truth.

By Modus Ponens, (1) and 'not A or B' entail 'if A then B'.

So 'not A or B' entails 'if A then B'.

Also, 'if A then B' entails 'not A or B'.

## The logic of material conditionals

		$A \rightarrow B$
Modus Ponens	if A then B, A ∴ B	valid
Conditional Proof	A entails B : . if A then B	valid
Or-to-If	A or B∴ if not ¬A then B	valid
Import-Export	if A then if B then C ∴ if A and B then C	valid
Contraposition	if A then B :. if not B then not A	valid
Transitivity	if A then B, if B then C ∴ if A then C	valid
SDA	if A or B then C if A then C and if B then C	valid
Antec. Strength.	if A then C ∴ if A and B then C	valid
False Antec.	not A :. if A then B	valid
True Cons.	B∴ if A then B	valid

1. True Cons. B: if A then B

The lecture ends at 2 pm. Therefore: If the building collapses at 1.45 then the lecture ends at 2 pm.

- False Antec. not A ∴ if A then B
  It is not the case that if it will rain tomorrow then the Moon will fall onto the Earth. Therefore: It will rain tomorrow.
- Antec. Strength. if A then C ∴ if A and B then C
  If you add sugar to your coffee, it will taste good. Therefore: If you add
  sugar and vinegar to your coffee, it will taste good.
- Contraposition if A then B ∴ if not B then not A
  If our opponents are cheating, we will never find out. Therefore: If we will find out that our opponents are cheating, then they aren't cheating.

Russell and Whitehead, Principia Mathematica (1913):

...if p and  $\neg p \lor q$  are both true, then q is true. In this sense, the proposition  $\neg p \lor q$  will be quoted as stating that p implies q. (p.7)

So 'the building collapses at 1.45' implies 'the lecture ends at 2 pm'.

C.I. Lewis (1918):

- $\neg p \lor q$  is not a good formalization of 'p implies q'.
- A better one is  $\Box(p \rightarrow q)$ .

Some have argued that  $\Box(p \rightarrow q)$  is also a good formalization of 'if p then q'.

Define  $A \rightarrow B$  as  $\Box(A \rightarrow B)$ .

Kripke semantics for  $\neg$ 

If  $M = \langle W, R, V \rangle$  is a Kripke model, then

 $M, w \models A \neg B$  iff for all v such that  $wRv, M, v \not\models A$  or  $M, v \models B$ .

What is R?

- If Oswald did not kill Kennedy then someone else did.
- $\Box(p \rightarrow q)$

Hypothesis: *wRv* iff *v* is compatible with what is known at *w*.

Modus Ponens is valid because epistemic accessibility is reflexive.

- Suppose  $\Box(A \rightarrow B)$  and A.
- $\Box(A \rightarrow B)$  entails  $A \rightarrow B$ .
- $A \rightarrow B$  and A entail B.
- So B.

		$A \rightarrow B$	A ⊰ B
Modus Ponens	if A then B, A ∴ B	valid	valid
Conditional Proof	A entails B : . if A then B	valid	valid
Or-to-If	$A \lor B \therefore$ if not A then B	valid	invalid
Import-Export	if A then if B then C ∴ if A and B then C	valid	invalid
Contraposition	if A then B .:. if not B then not A	valid	valid
Transitivity	if A then B, if B then C ∴ if A then C	valid	valid
SDA	if A or B then C : if A then C and if B then C	valid	valid
Antec. Strength.	if A then C ∴ if A and B then C	valid	valid
False Antec.	not A ∴ if A then B	valid	invalid
True Cons.	B∴if A then B	valid	invalid

#### **Problems**:

•  $A \rightarrow B \models \neg B \rightarrow \neg A$ 

If our opponents are cheating, we will never find out. Therefore: If we will find out that our opponents are cheating, then they aren't cheating.

• 
$$A \rightarrow B \models (A \land C) \rightarrow \neg B$$

If you add sugar to your coffee, it will taste good. Therefore: If you add sugar and vinegar to your coffee, it will taste good.

•  $A \rightarrow B, B \rightarrow C \models A \rightarrow C$ .

If I quit my job, I won't be able to pay rent. If I win a million, I'll quit my job. Therefore: if I win a million, I won't be able to pay rent.

#### Possible response:

The accessibility relation depends on conversational context.

- 'If you add sugar to your coffee, it will taste good.'
  - Here worlds where you add sugar and vinegar to your coffee are ignored/inaccessible.
- 'If you add sugar and vinegar to your coffee, it will taste good.'
  - Now these worlds are no longer ignored/inaccessible.

#### Another problem

Why are we often unsure about conditionals?

• I'm not sure whether NATO will attack Russia if Russia invades Estonia.

This is not because I'm unsure about what I know.