

# Logic 2: Modal Logic

## Lecture 12

---

Wolfgang Schwarz

University of Edinburgh

# Review

---

## Kripke semantics for obligation and permission

$M, w \models OA$  iff  $M, v \models A$  for all  $v$  with  $wRv$ .

$M, w \models PA$  iff  $M, v \models A$  for some  $v$  with  $wRv$ .

What is  $R$ ?

- $wRv$  iff all norms are respected at  $v$ .
- $wRv$  iff all norms at  $w$  are respected at  $v$ .

Assuming seriality, the resulting deontic logic is either KD45 or D.

# Redefining Accessibility

---

### The Bank Robber Paradox

Mary robbed a bank.

1. Mary ought to go to jail.  $Oj$
2. Mary ought to not have robbed the bank.  $O \neg r$
3. But not:  $O(j \wedge \neg r)$ .

In an ideal world, Mary didn't rob the bank and doesn't go to jail.

Assuming  $wRv$  iff  $v$  is ideal (relative to  $w$ ), then  $Oj$  is false.

Why is it true that Mary ought to go to jail?

Because she has robbed the bank, and this is settled.

Among all worlds, the best worlds are worlds where Mary didn't rob the bank.

Among the open worlds, the best worlds are worlds where Mary goes to jail.

New definition of accessibility:

$wRv$  iff  $v$  is one of the best worlds (by  $w$ 's norms) among those that are open at  $w$ .

Obligations depend not just on the norms, but also on the circumstances.

- Circumstance: Mary has robbed a bank.
- Norm: Anyone who has robbed a bank must go to jail.
- So: Mary must go to jail.

## Redefining Accessibility

New definition of accessibility:

$wRv$  iff  $v$  is one of the best worlds (by  $w$ 's norms) among those that are open at  $w$ .

Obligations depend not just on the norms, but also on the circumstances.

- Circumstance: You walk past a drowning baby.
- Norm: Don't let babies drown.
- So: You must rescue the baby.

## Redefining Accessibility

Now we have a problem with “Mary ought not to have robbed the bank”.

Among the open worlds, there are none in which Mary didn't rob the bank.

Here we consider worlds that aren't open.

Revised new definition of accessibility:

$wRv$  iff  $v$  is one of the best worlds (by  $w$ 's norms) among those that are circumstantially possible at  $w$ .

What is circumstantially possible depends on conversational context.

### The Bank Robber Paradox

1. Mary ought to go to jail.  $Oj$
2. Mary ought to not have robbed the bank.  $O \neg r$
3. But not:  $O(j \wedge \neg r)$ .

Different circumstances are relevant in 1 and 2.

If we don't equivocate, either  $Oj$  or  $O \neg r$  is false.

### The Samaritan Paradox

1. Jones ought to help the injured Smith.
2. That Jones helps the injured Smith entails that Smith has been injured.
3. But not: Smith ought to have been injured.

Different circumstances are (pragmatically) held fixed in 1 and 3.

### Professor Procrastinate

1. Professor Procrastinate ought not to accept the review.
2. Professor Procrastinate ought to accept and complete the review.

Different circumstances are held fixed in 1 and 2.

## Deontic conditionals

---

### The Gentle Murder Paradox (Forrester 1984)

- John will kill his mother.
- If John will kill his mother, he should kill her gently.
- John ought to not kill his mother, gently or otherwise.
  
- $k$
- $k \rightarrow O g$
- $O \neg g$

1-3 entail  $O g$  and  $O \neg g$ !

### The Gentle Murder Paradox (Forrester 1984)

- John will kill his mother.
- If John will kill his mother, he should kill her gently.
- John ought to not kill his mother, gently or otherwise.
  
- $k$
- $O(k \rightarrow g)$
- $O \neg k$

But  $O \neg k \models_K O(k \rightarrow p)$ , for any  $p$ .

So 3 would entail 'If John will kill his mother, he should kill her brutally.'

(\*) If John will kill his mother, he should kill her gently.

## Deontic conditionals

(\*) If John will kill his mother, he should kill her gently.

Intuitively: (\*) says that at the best worlds among those at which John kills his mother, he kills her gently.

This cannot be expressed in  $\mathcal{L}_M$ .

But we can add a **binary** obligation operator  $O(\cdot/\cdot)$  to express it:  $O(q/p)$ .

$O(q)$  is true iff  $q$  is true at the best worlds at which relevant circumstances obtain.

$O(q/p)$  is true iff  $q$  is true at the best worlds at which relevant circumstances obtain and  $p$  is the case.

# Neighbourhood Semantics

---

Let's drop the assumption that  $\Box$  and  $\Diamond$  are quantifiers over worlds.

For every sentence  $A$  there is a set of worlds  $[A]$  at which  $A$  is true.

We assume that the box and the diamond express properties of these sets.

$\Box A$  is true iff  $[A]$  is required/necessary/known/etc.  $\Box A$  is true at  $w$  iff  $[A]$  is required/necessary/known/etc at  $w$ .  $\Diamond A$  is true at  $w$  iff  $[A]$  is in the "neighbourhood" of  $w$ .

A **neighbourhood model** consists of

- a non-empty set  $W$ ,
- a function  $N$  that assigns to each member of  $W$  a set of subsets of  $W$ , and
- a function  $V$  that assigns to each sentence letter of  $\mathcal{L}_M$  a subset of  $W$ .

### Neighbourhood semantics

$M, w \models \Box A$  iff  $[A]^M$  is in  $N(w)$ .

$M, w \models \Diamond A$  iff  $[\neg A]^M$  is not in  $N(w)$ .

The set of sentences that are true at all worlds in all neighbourhood models is called **system E**. It is axiomatized by

(Dual)  $\neg\Box A \leftrightarrow \Diamond\neg A$

(CPL) Any truth-functional consequence of sentences in the system is in the system.

(RE) If  $A \leftrightarrow B$  is in the system then so is  $\Box A \leftrightarrow \Box B$ .

(K), (T), (D), (B), (4), (5), and (G) are not valid in neighbourhood semantics. Nor are

- $\Box p \wedge \Box q \rightarrow \Box(p \wedge q)$
- $\Box(p \vee \neg p)$

# Pragmatics

---

## The Paradox of Free Choice (von Wright 1967)

Intuitively,

- You may have beer or wine

entails

- You are permitted to have beer, and
- You are permitted to have wine.

But  $P(A \vee B) \not\vdash_K P(A)$  and  $P(A \vee B) \not\vdash_E P(A)$ .

To allow this inference, we would need to give up (RE).

(1) Amy is at the pub or in the library.

↗ (1a) Amy might be at the pub.

↗ (1b) Amy might be in the library.

This is not an entailment, but an **implicature**.

- It would (normally) be uncooperative to assert (1) unless (1a) and (1b) were true.
- So we take an assertion of (1) to indicate the truth of (1a) and (1b).

(1) You may have beer or wine.

↗ (1a) You may have beer.

↗ (1b) You may have wine.

A Sign that this is an implicature:

- 'You may not have beer' does not imply 'You may not have beer or wine'.