Logic 2: Modal Logic

Lecture 11

Wolfgang Schwarz

University of Edinburgh

Deontic logics

Deontic logics

Deontic logic formalizes reasoning about norms.

- Obligation
- Permission
- Prohibition
- Optionality
- Rights
- Duties
- Supererogation
- etc.

Deontic logics

We focus on two operators:

- O: It is obligatory/required that ...
- P: It is permitted that ...

You must return the library book

$\Rightarrow$ It is required that you return the library book

$\Rightarrow \mathrm{O}\, p$

Deontic logics

Can we give a possible-worlds analysis for O and P?

It is obligatory that *p* iff *p* is true at all worlds ...

It is obligatory that *p* iff *p* is true at all worlds where the norms are fulfilled.

Call a world **ideal** if it contains no violations of any (relevant) norms.

Deontic logics

A simple absolutist Kripke semantics

$M, w \models \mathrm{O}A$ iff $M, v \models A$ for all v with wRv.
$M, w \models \mathrm{P}A$ iff $M, v \models A$ for some v with wRv.

wRv iff v is ideal.

- Is R reflexive (for every w, wRw)?
- Is R serial (for every w there is some v such that wRv)?
- Is R transitive (if wRv and vRu then wRu)?
- Is R symmetric (if wRv then vRw)?
- Is R euclidean (if wRv and wRu then vRu)?

Assuming seriality, we get the logic KD45.

Deontic logics

KD45 is axiomatized by

- (**K**) $\Box(A \to B) \to (\Box A \to \Box B)$
- (**D**) $\Box A \to \Diamond A$
- (**4**) $\Box A \to \Box\Box A$
- (**5**) $\Diamond A \to \Box\Diamond A$

and the rules (CPL) and (Nec).

Deontic logics

We have assumed that what is required does not depend on what is the case: the same worlds are ideal relative to every world.

How could what is required depend on what is the case?

- Norms depend on non-normative circumstances.
- Instances of norms are sensitive to circumstances.

A simple relativist Kripke semantics

$M, w \models \mathrm{O}A$ iff $M, v \models A$ for all v with wRv.
$M, w \models \mathrm{P}A$ iff $M, v \models A$ for some v with wRv.

wRv iff v is ideal relative to the norms of w.

- Is R reflexive (for every w, wRw)?
- Is R serial (for every w there is some v such that wRv)?
- Is R transitive (if wRv and vRu then wRu)?
- Is R symmetric (if wRv then vRw)?
- Is R euclidean (if wRv and wRu then vRu)?

Assuming seriality, we get the **standard deontic logic** D.

Challenges to normal deontic logics

Challenges to normal deontic logics

1. Obligatory tautologies

If A is true at all worlds, then $\mathsf{O}A$ is true at all worlds.

$\models_K \mathsf{O}(p \vee \neg p)$

But are you obligated to either go to class or not go to class?

Challenges to normal deontic logics

2. No scenarios without norms (Chellas 1980)

If there are no norms, then there are no obligations or permissions.

It is not a logical truth that there are norms.

$\not\models_K \mathsf{O}(p \vee \neg p)$
$\not\models_K \mathsf{P}(p \vee \neg p)$

3. Conflicting obligations (Lemmon 1962)

You may be obligated to do p and obligated to do $\neg p$, without being obligated to do everything.

$(\mathsf{O}\, p \wedge \mathsf{O}\, \neg p) \models_K \mathsf{O}\, q$.

4. The Samaritan Paradox (Prior 1958)

Smith has been robbed and injured.

- Jones ought to help the injured Smith.
- That Jones helps the injured Smith entails that Smith has been injured.

If $A \models_K B$, then $\mathrm{O}A \models_K \mathrm{O}B$.

- So: Smith ought to have been injured?!

Challenges to normal deontic logics

5. The Knowledge Paradox (Aqvist 1967)

- Jones ought to know that there is a fire.
- That Jones knows that there is a fire entails that there is a fire.
- So there ought to be a fire?

6. The Bank Robber Paradox

Mary robbed a bank.

- Mary ought to go to jail.
- Mary ought to not have robbed the bank.

$\mathrm{O}A \wedge \mathrm{O}B \models_K \mathrm{O}(A \wedge B)$

- So: it ought to be the case that Mary didn't rob the bank and yet she goes to jail?

Challenges to normal deontic logics

7. Professor Procrastinate (Pargetter and Jackson 1986)

- Professor Procrastinate ought not to accept the review.
- Professor Procrastinate ought to accept and complete the review.

$\mathrm{O}(A \wedge B) \models_K \mathrm{O}A$

8. Ross's Paradox (Ross 1943)

Intuitively,

- you must either mail or burn the letter

entails

- you are permitted to mail the letter, and
- you are permitted to burn the letter.

$\mathsf{O}\, m \models_K \mathsf{O}(m \vee b)$

Challenges to normal deontic logics

9. The Paradox of Free Choice (von Wright 1967)

Intuitively,

- you may have beer or wine

entails

- you are permitted to have beer, and
- you are permitted to have wine.

But $\mathrm{P}\, b \not\models_K \mathrm{P}(b \vee w)$.

Challenges to normal deontic logics

10. The Gentle Murder Paradox (Forrester 1984)

- John ought to not buy meat. $\mathrm{O}\neg p$
- If he does buy meat, he should buy meat from sustainable sources. $p \rightarrow \mathrm{O}\, q$
- John does buy meat. p

By modus ponens, we can infer $\mathrm{O}\, q$.

Also, since q entails p, we get $\mathrm{O}\, p$.

Challenges to normal deontic logics

10. The Gentle Murder Paradox (Forrester 1984)

- John ought to not buy meat. $\mathrm{O}\neg p$
- If he does buy meat, he should buy meat from sustainable sources. $\mathrm{O}(p \rightarrow q)$
- John does buy meat. p

Now we can no longer infer $\mathrm{O}\, q$.

But $\mathrm{O}\neg p \models_K \mathrm{O}(p \rightarrow r)$.

So $\neg\, \mathrm{O}\, p$ entails

- If John does buy meat, he should buy from factory farms. $\mathrm{O}(p \rightarrow r)$

11. The Miners Puzzle (Kolodny and MacFarlane 2010)

- If the miners are in shaft A, we ought to block shaft A.
- If the miners are in shaft B, we ought to block shaft B.
- We ought to block neither shaft.

- $s_A \vee s_B$
- $s_A \rightarrow \mathrm{O}\, b_A$
- $s_B \rightarrow \mathrm{O}\, b_B$
- $\mathrm{O}(\neg b_A \wedge \neg b_B)$

These are inconsistent in K.

11. The Miners Puzzle (Kolodny and MacFarlane 2010)

- If the miners are in shaft A, we ought to block shaft A.
- If the miners are in shaft B, we ought to block shaft B.
- We ought to block neither shaft.

- $s_A \vee s_B$
- $\mathrm{O}(s_A \rightarrow b_A)$
- $\mathrm{O}(s_B \rightarrow b_B)$
- $\mathrm{O}(\neg b_A \wedge \neg b_B)$

These K-entail

- $\mathrm{O}(\neg s_A \wedge \neg s_B \wedge \neg b_A \wedge \neg b_B)$