Logic 2: Modal Logic

Lecture 9

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A model of knowledge

A model of knowledge

In epistemic logic, the box represents knowledge.

Possible-worlds analysis of knowledge

S knows that *P* iff *P* is true at all worlds compatible with *S*'s knowledge.

In epistemic Kripke models, wRv means v is compatible with the agent's knowledge at w.

A model of knowledge

More knowledge = fewer open possibilities

The duke has been murdered. There are four suspects: the gardener, the butler, the cook, and the maid.

A model of knowledge

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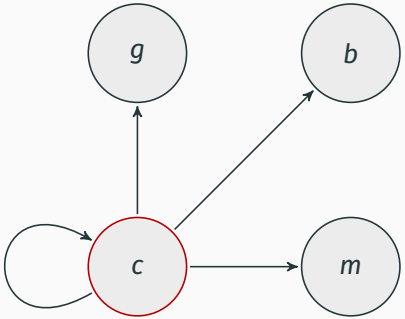
The gardener has an alibi.

A model of knowledge

More knowledge = fewer accessible worlds

The cook has murdered the duke. The detective investigates.



A model of knowledge

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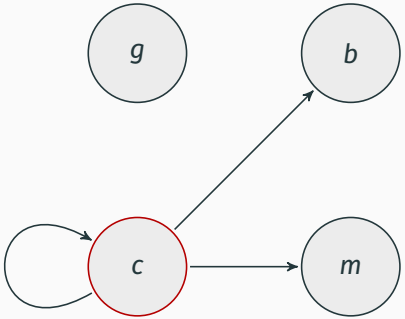

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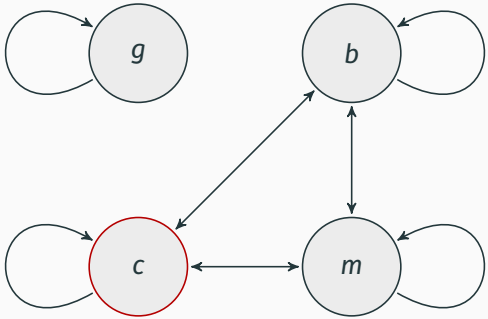


A model of knowledge

Kripke semantics

$M, w \models \Box A$ iff $M, v \models A$ for all v such that wRv.
$M, w \models \Diamond A$ iff $M, v \models A$ for some v such that wRv.

In epistemic logic, we usually write the box as 'K'.

I write the diamond as 'M'.

Kripke semantics

$M, w \models \mathrm{K}A$ iff $M, v \models A$ for all v such that wRv.
$M, w \models \mathrm{M}A$ iff $M, v \models A$ for some v such that wRv.

Logical Omniscience

Logical Omniscience

A consequence of our semantics:

- Knowledge is **closed under known consequence**: $\mathsf{K}A, \mathsf{K}(A \to B) \models \mathsf{K}B$.
- Knowledge is **closed under logical consequence**: If $A \models B$ then $\models \mathsf{K}A \to \mathsf{K}B$.

This seems wrong.

Response 1: Our semantics is only adequate for ideal agents.

Response 2: We are modelling (a tidied-up concept of) **implicit knowledge**.

Logical Omniscience

Fred Dretske's (1970) argument against (K):

1. I know that I have hands. $\mathsf{K}\,p$
2. I know that if I have hands then I'm not a brain in a vat. $\mathsf{K}(p \rightarrow \neg q)$
3. I do not know that I'm not a brain in a vat. $\neg \mathsf{K} \neg q$

$\models \mathsf{K}(A \rightarrow B) \rightarrow (\mathsf{K}\,A \rightarrow \mathsf{K}\,B)$

$\mathsf{K}(A \rightarrow B), \mathsf{K}\,A \models \mathsf{K}\,B$

Epistemic Accessibility

Epistemic Accessibility

A world *v* is epistemically accessible for an agent at *w* (*wRv*) iff

- the agent's knowledge at *w* is compatible with the hypothesis that *v* is the actual world;
- *v* might be the actual world, for all the agent knows;
- whatever the agent knows at *w* is true at *v*.

Can we be more informative?

Epistemic Accessibility

Proposal 0

A world v is epistemically accessible for an agent at w (wRv) iff the agent's evidence at w is compatible with v.

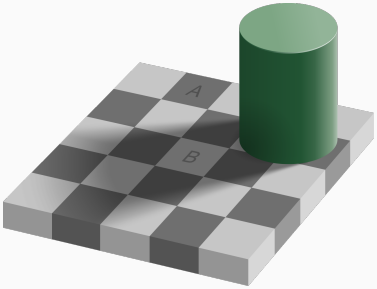
An agent's evidence is what her senses tell her.

Epistemic Accessibility

Proposal 1

A world v is epistemically accessible for an agent at w (wRv) iff the agent's evidence at w is compatible with v.

An agent's evidence is what her senses and memory tell her.



My senses tell me that square A is darker than square B.

But square A is not darker.
And I don't believe that square A is darker.

$\mathrm{K}\,p \rightarrow p$ would become invalid.
$\mathrm{K}\,p \rightarrow \mathrm{B}\,p$ would become invalid.

Epistemic Accessibility

Proposal 2

A world v is epistemically accessible for an agent at w (wRv) iff the agent's sense experiences and memory at v are the same as at w.

- Is R reflexive? (For all w, wRw)
- Is R transitive? (If wRv and vRu then wRu)
- Is R symmetric? (If wRv then vRw)

Yes. R is an equivalence relation.

Epistemic Accessibility

Proposal 2

A world v is epistemically accessible for an agent at w (wRv) iff the agent's sense experiences and memory at v are the same as at w.

We get an S5 logic.

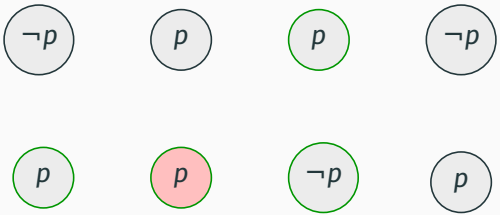
- (**K**) $\mathrm{K}(A \rightarrow B) \rightarrow (\mathrm{K}A \rightarrow \mathrm{K}B)$
- (**T**) $\mathrm{K}A \rightarrow A$
- (**4**) $\mathrm{K}A \rightarrow \mathrm{KK}A$
- (**B**) $A \rightarrow \mathrm{KM}A$
- (**5**) $\mathrm{M}A \rightarrow \mathrm{KM}A$

Epistemic Accessibility

Proposal 2

A world v is epistemically accessible for an agent at w (wRv) iff the agent's sense experiences and memory at v are the same as at w.

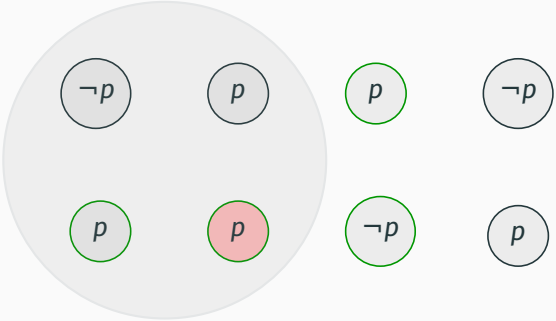
We also get scepticism about the external world.



Epistemic Accessibility

Proposal 3 (Lewis 1996)

A world v is epistemically accessible for an agent at w (wRv) iff the agent's sense experiences and memory at v are the same as at w and v is not properly ignored



Reflexivity, Seriality, Symmetry, Transitivity, Euclidity

Reflexivity, Seriality, Symmetry, Transitivity, Euclidity

Almost everyone wants the logic of knowledge to validate

(**T**) $\mathsf{K}A \rightarrow A$

So R should be reflexive. We then automatically get

(**D**) $\mathsf{K}A \rightarrow \mathsf{M}A$

Reflexivity, Seriality, Symmetry, Transitivity, Euclidity

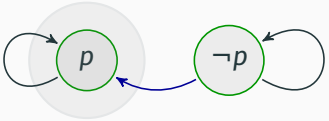
Should R be symmetric? Do we want (B) to come out valid?

(**B**) $A \to \mathrm{KM}\, A$

Suppose you falsely believe $\neg p$.

- p is true.
- You believe that you know $\neg p$.
- You don't believe that you don't know $\neg p$.
- You don't know that you don't know $\neg p$.
- $\mathrm{K} \neg \mathrm{K} \neg p$ is false.
- $\mathrm{KM}\, p$ is false.

Also, this would lead to skepticism.



Reflexivity, Seriality, Symmetry, Transitivity, Euclidity

Positive Introspection:

(**4**) $\mathsf{K}A \rightarrow \mathsf{KK}A$

Negative Introspection:

(**5**) $\mathsf{M}A \rightarrow \mathsf{KM}A$

(5) corresponds to euclidity. Euclidity and reflexivity entail symmetry. So philosophers mostly reject (5).

(4) corresponds to transitivity. It is controversial.