Logic 2: Modal Logic

Lecture 1

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Course info

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- This is an intermediate logic course focusing on modal logic. You will learn about
  - the syntax and semantics of modal propositional and predicate logic
  - different proof methods: axiomatic calculi and tableaux methods
  - the role of models in logic
  - soundness and completeness of proof methods
  - epistemic logic, deontic logic, temporal logic, provability logic
  - the interpretation of modals and conditionals in natural language
  - and much more
- You must have taken Logic 1 or something equivalent.
- Logic 1 was easy. This course will be harder.

Course info

Website, Readings, Exercises

- The website for this course is *www.wolfgangschwarz.net/logic2*.
- Each week I will post extensive lecture notes with exercises.
- Please read the notes and do the exercises.

Course info

Tutorials

- Tutorials run for two hours.
- In the first hour, we go through the answers to selected exercises.
- The second hour is an open Q&A session, often with more answers to exercises.
- Only the first hour is compulsory.

Logic 2 Lab

- Time and Location TBD
- Do the exercises, get help, talk to others, ...

Course info

Assessment

- 20% First take-home test (probably 16/10–19/10)
- 30% Second take-home test (probably 20/11–23/11)
- 50% Final exam (sometime in December)

What is modal logic?

What is modal logic?

In propositional logic, we can formalize arguments like this:

It is either raining or snowing.	$r \vee s$
It is not snowing.	$\neg s$
It is raining.	r

Remember: $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$

We can show that the argument is valid.

What is modal logic?

We can also formalize this argument:

All birds have feathers.	p
All penguins are birds.	q
All penguins have feathers.	r

But in the language of propositional logic we can't bring out why the argument is valid.

What is modal logic?

We need to **extend our formal language**.

All birds have feathers.	$\forall x(Bx \rightarrow Fx)$
All penguins are birds.	$\forall x(Px \rightarrow Bx)$
All penguins have feathers.	$\forall x(Px \rightarrow Fx)$

Remember: $\forall$, $\exists$

We can now show that the argument is valid.

What is modal logic?

Another argument:

It is possible that it is raining.	p	p
It is certain that we will get wet if it is raining.	q	$r \rightarrow q$
It is possible that we will get wet.	r	s

What is modal logic?

Every argument of this form is plausibly valid:

It is possible that *A*.
It is certain that *B* if *A*.

It is possible that *B*.

What is modal logic?

We need to **extend our formal language**.

It is possible that it is raining.	$\Diamond r$
It is certain that we will get wet if it is raining.	$\Box(r \rightarrow w)$
It is possible that we will get wet.	$\Diamond w$

$\Diamond$ translates ‘it is possible that ...’.

$\Box$ translates ‘it is certain that ...’.

What is modal logic?

Another argument:

You are allowed to come in.	$\mathrm{P}\,c$
You must take off your shoes if you come in.	$\mathrm{O}(c \rightarrow s)$
You are allowed to take off your shoes.	$\mathrm{P}\,s$

P translates ‘it is permitted that ...’.

O translates ‘it is obligatory that ...’

What is modal logic?

Modal logic is used to formalise reasoning about

- possibility and necessity
- permission and obligation
- knowledge and ignorance
- past, present, and future
- provability in mathematical theories
- the processing of computer programs
- and many other topics

The box $\Box$ *often* means ‘it is necessary that’ and the diamond $\Diamond$ ‘it is possible that’, but these expressions in turn can mean different things.

Translating from English

Translating from English

- John can’t leave the room.
- $\neg\Diamond p$ (p: John leaves the room)
- It is not possible that John leaves the room.

- John mustn’t leave the room.
- $\Box\neg p$ or $\mathrm{O}\,\neg p$
- It is necessary/obligatory that John does not leave the room.

Translating from English

- Going to lectures is no guarantee that you'll do well in the exam.
- $\neg\Box(g \rightarrow w)$ (g: You go to the lectures, w: You do well in the exam)
- It is not necessary/certain that if you go to lectures you will do well in the exam.

Translating from English

- If the lights are on, Ada might be in her office.
- $\Diamond(l \wedge o)$ (l: The lights are on, o: Ada is in her office)
- It is possible that the lights are on and Ada is in her office.