Chapter 9

Exercise 9.1

(a) $Srj \wedge Skj$; r: Keren, k: Keziah, j: Jemima, S: – is a sister of –
(b) $\forall x(Mx \to Ox)$; M: – is a myriapod, O: – is oviparous
(c) $\exists x(Cx \wedge Nx \wedge Hfx)$; f: Fred, C: – is a car, N: – is new, H: – has –
(d) $\neg\forall x(Sx \to Lxl)$; l: logic; S: – is a student, L: – loves –
(e) $\forall x((Sx \wedge Lxl) \to \exists yLxy)$; l: logic; S: – is a student, L: – loves –

Exercise 9.2

Let the model M be given by $D = \{\text{Rome}, \text{Paris}\}$ and $V(F) = \{\text{Rome}\}$. By clause (a) of definition 9.2, $M, g' \models Fx$ holds for every assignment function g' that maps x to Rome, because then $g'(x) \in V(F)$. By clause (h) it follows that $M, g \models \exists xFx$ for every assignment function g. By clause (a) again, $M, g' \not\models Fx$ for every assignment function g' that maps x to Paris. By clause (g), it follows that $M, g \not\models \forall xFx$ for every assignment function g. So $\exists xFx$ is true (in M) relative to every assignment function while $\forall xFx$ is false relative to every assignment function. By clause (e) it follows that $\exists xFx \to \forall xFx$ is false in M relative to every assignment function.

Exercise 9.3

For both cases, use Fx as the sentence A, and $\neg Fx$ as B, and consider a model in which F applies to some but not to all individuals. Both Fx and $\neg Fx$ are then true relative to some assignment functions and false relative to others. So neither sentence is true in the model. But $Fx \vee \neg Fx$ is true relative to every assignment function.

Exercise 9.4

There are many non-reflexive models in which $\Box p \to p$ is true at some world – for example, any non-reflexive model in which p is false at all worlds.

For the more general question, let M_1 be a model with a single world that can see itself. Let M_2 be a model with two worlds, each of which can see the other but not itself. In both models, all sentence letters are false at all worlds. The very same $\mathfrak{L}_M$-sentences are true at all worlds in these models (as a simple proof by induction shows). But the first model is reflexive and the second isn't. So there is no $\mathfrak{L}_M$-question that is true at a world in a model iff the model's accessibility relation is reflexive.

Exercise 9.5

Use umsu.de/trees/.

Exercise 9.6

If a sentence is valid (in first-order predicate logic) then a fully expanded tree for the sentence will close and show that the sentence is valid. But if a sentence is not valid, the tree might grow forever. There is no algorithm for detecting whether a tree will grow forever.

Exercise 9.7

(a) $\Box Fa$
a: John, F: – is hungry.
(Might be classified as either *de re* or *de dicto*.)

(b) $\Box\forall x(Fx \to Gx)$
F: – is a cyclist, G: – has legs.
This is *de dicto*. Also correct (but different in meaning) is the *de re* translation $\forall x(Fx \to \Box Gx)$. Close but incorrect (and *de re*): $\forall x\Box(Fx \to Gx)$.

(c) $\forall x(Fx \to \Diamond Gx)$
F: – is a day, G: – is our last day.
Better: $\forall x(Fx \to \Diamond(Hx \land \neg\exists y(Fy \land Lyx \land Hy)))$
F: – is a day, L: – is later than –, H: We are alive on –.
Both *de re*. The English sentence could also be understood *de dicto*, as $\Diamond\forall x(Fx \to Gx)$, but that would be a very strange thing to say.

(d) $\forall x\, \mathsf{O}(Fx \to Gx)$
F: – wants to leave early, G: – leaves quietly.
Even better, if we can use the conditional obligation operator: $\forall x\, \mathsf{O}(Gx/Fx)$.
These aren't too far off either: $\forall x(Fx \to \mathsf{O}\, Gx)$, $\mathsf{O}\,\forall x(Fx \to Gx)$.

All of these are *de re*.

(e) $\forall x(\exists y(Fy \land Hxy) \to \mathsf{P}\,Gx)$
F: – is a ticket, G: – enters, H: – bought –.

Perhaps even better: $\forall x\,\mathsf{P}(Gx/\exists y(Fy \land Hxy))$. Both of these are *de re*.

You could translate 'bought a ticket' as a simple predicate here; you could also use a temporal operator to account for the past tense of 'bought' (but it's confusing to use two different kinds of 'P' in one sentence).

Exercise 9.8

See the previous answer.

Exercise 9.9

Use umsu.de/trees/.

Exercise 9.10

We assume that some branch on a tree contains nodes $b = c$ and A. We have to show that we can add $A[b//c]$ without using the second version of Leibniz' Law.

k.	$b = c$	
n.	A	
m.	$b = b$	(SI)
m+1.	$c = b$	(k, m, LL (first version))
m+2.	$A[b//c]$	(m+1, n, LL (first version))

Exercise 9.11

(a)

1.	$a = a$	(SI)
2.	$\forall x\, x \neq a \to a \neq a$	(UI)
3.	$\neg\forall x\, x \neq a$	(1, 2, CPL)
4.	$\neg\exists x\, x = a \leftrightarrow \forall x\, x \neq a$	$(\forall\exists)$
5.	$\exists x\, x = a$	(3, 4, CPL)
6.	$\Box\exists x\, x = a$	(5, Nec)

(b) There are many correct answers. For example: historians debate whether Homer ever existed. If a translates ‘Homer’ then $\exists x\, x = a$ is arguably false if Homer isn’t a real person. Since the available evidence is compatible with $\neg\exists x\, x = a$, the sentence $\Box\exists x\, x = a$ is false on an epistemic interpretation of the box.

Where does the proof go wrong? Each of steps 1, 2, and 6 might be blamed.

Exercise 9.12

(a) $\exists x\exists y(Fx \land Fy \land x \neq y \land \forall z(Fz \to (z = x \lor z = y)))$
(b) $\forall x\forall y\forall z\forall v(Fx \land Fy \land Fz \land Fv \to (x = y \lor x = z \lor x = v \lor y = z \lor y = v \lor z = v))$

Exercise 9.13

The *de dicto* reading of (a) can be translated as

$$\Diamond\exists x(Px \land \forall y(Py \to x = y) \land x = c),$$

where ‘P’ translates ‘– is 45th US President’ and ‘c’ denotes Hillary Clinton. The *de re* reading can be translated as

$$\exists x(Px \land \forall y(Py \to x = y) \land \Diamond x = c).$$

The answers to (b) and (c) are analogous.