Chapter 1

Exercise 8.1

(E1)–(E5) are invalid assuming that ‘if A then B’ is true iff both A and B are true. There are, of course, strong reasons against the analysis of English conditionals as conjunctions.

Exercise 8.2

For example: $\neg A \dashv A$ or $(A \vee \neg A) \dashv A$.

Exercise 8.3

$W = \{w\}, R = \emptyset, V(p) = \{w\}, V(q) = \emptyset$.

Exercise 8.4

Use umsu.de/trees/.

Exercise 8.5

(E1)–(E5) all work equally well in the subjunctive mood. For (E4) and (E5):

- If our opponents had been cheating, we would never have found out. Therefore: If we had found out that our opponents are cheating, then they wouldn’t have been cheating.
- If you had added sugar to your coffee, it would have tasted good. Therefore: If you had added sugar and vinegar to your coffee, it would have tasted good.

Both of these inferences are valid if subjunctive conditionals are strict conditionals. But they don’t sound good.

Exercise 8.6

Suppose $A \to B$ is assertable. Then $A \to B$ is known. So $\mathsf{K}(A \to B)$. In S4, it follows that $\mathsf{K}\,\mathsf{K}(A \to B)$. So the epistemically strict conditional $\mathsf{K}(A \to B)$ is assertable. Conversely, if $\mathsf{K}(A \to B)$ is assertable, then it is known; so $\mathsf{K}\,\mathsf{K}(A \to B)$. In S4, it follows that $\mathsf{K}(A \to B)$. So $A \to B$ is assertable.

Exercise 8.7

The 'or-to-if' inference is not valid on the assumption that the conditional is epistemically strict. For example, if p and q are both true at the actual world and both false at some epistemically accessible world, then 'p or q' is true but 'if p then q' is false (on the strict analysis).

The inference might nonetheless look reasonable because it would normally be inappropriate to assert a disjunction 'p or q' unless the disjunction is known – unless it is true at all epistemically accessible worlds. And if $p \vee q$ is true at all epistemically accessible worlds then $\neg p \to q$ is also true at all epistemically accessible worlds, and so $\Box(p \to q)$ is true. Thus the conclusion of or-to-if is true in any situation in which the premise is *assertable*. If the logic of knowledge validates the (4)-schema, we can go further and say that the conclusion is assertable in any situation in which the premise is assertable.

Exercise 8.8

Assume that R is asymmetric and quasi-connected. We want to show that R is transitive. So assume we have xRy and yRz. By quasi-connectedness, yRz implies that either yRx or xRz. By asymmetry, we can't have yRx, since we have xRy. So xRz.

Exercise 8.9

We have the following equivalences (using '$\Leftrightarrow$' to mean that the expressions on either side are equivalent):

$$u \npreceq_w v \Leftrightarrow \neg(u \preceq_w v) \Leftrightarrow \neg(v \nprec_w u) \Leftrightarrow v \prec_w u.$$

So you can simply replace every instance of $\omega \prec_w v$ in the conditions by $v \npreceq_w \omega$, and every instance of $\omega \nprec_w v$ by $v \preceq_w \omega$.

Asymmetry thereby turns into: if $u \npreceq_w v$ then $v \preceq_w u$. Equivalently: either $u \preceq_w v$ or $v \preceq_w u$. This property of relations is called **strong connectedness** or **completeness**. Notice that it entails reflexivity.

Quasi-connectedness turns into: if $u \npreceq_w v$ then for all t, either $t \npreceq_w v$ or $u \npreceq_w t$.

This is equivalent to transitivity for $\preceq$.

The Limit Assumption turns into: for any non-empty set of worlds X and world w there is a $v \in X$ such that there is no $u \in X$ with $v \not\preceq_w u$. Equivalently, for any non-empty set of worlds X and world w there is a $v \in X$ such that $v \preceq_w u$ for all $u \in X$.

Exercise 8.10

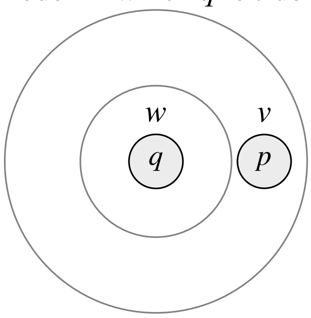
No. We don't want A and $\mathsf{O}(B/A)$ to entail B. Semantically, we don't want to assume that every world is among the best worlds relative to its own norms.

Exercise 8.11

Suppose $A \mathrel{\Box\!\!\rightarrow} B$ is true at some world w in some model M. So B is true at all the closest A-worlds to w. Now either A is true at w or A is false at w. If A is false at w, then $A \to B$ is true at w. If A is true at w, then w is one of the closest A-worlds to w, by Weak Centring; so B is true at w; and so $A \to B$ is true at w. Either way, then, $A \to B$ is true at w.

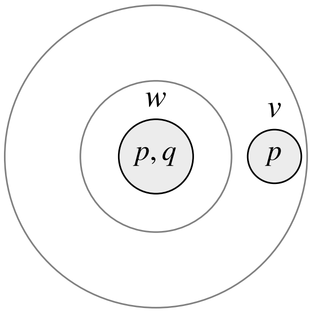
Exercise 8.12

(E1) is an inference from q to $p \mathrel{\Box\!\!\rightarrow} q$. To show that this is invalid, we need to give a model in which q is true at some world (w) while $p \mathrel{\Box\!\!\rightarrow} q$ is false (at w).



This model also shows that (E2) and (E3) are invalid. (E2) is an inference from $\neg p$ to $p \mathrel{\Box\!\!\rightarrow} q$. In the model, $\neg p$ is true at w but $p \mathrel{\Box\!\!\rightarrow} q$ is false. (E3) is an inference from $\neg(p \mathrel{\Box\!\!\rightarrow} q)$ to p. In the model, $\neg(p \mathrel{\Box\!\!\rightarrow} q)$ is true at w but p is false.

(E4) is an inference from $p \mathrel{\Box\!\!\rightarrow} q$ to $\neg q \mathrel{\Box\!\!\rightarrow} \neg p$. In the following model, the premise is true at w and the conclusion false.



Exercise 8.13

Frances has never learnt a foreign language, although she would have loved to learn French. If Frances had been given a choice between learning French and learning Italian, she would have chosen French. *If Frances had learned French or Italian then she would have learned French.* It does not follow that if Frances had learned Italian then she would have learned French.

The same style of example works for indicative conditionals.

Exercise 8.14

(a) Assume $A \land B$ is true at some world w in some model M. By Centring, w is among the closest A-worlds to w. By connectedness, w is the unique closest A-world to w. So B is true at all closest A-worlds to w.

(b) Assume $A \mathrel{\Box\!\!\rightarrow} (B \lor C)$ is true at some world w in some model M. So all the closest A-worlds to w are $(B \lor C)$-worlds. If there are no A-worlds then $A \mathrel{\Box\!\!\rightarrow} B$ and $A \mathrel{\Box\!\!\rightarrow} C$ are both true. If there are A-worlds then Stalnaker's semantics implies that there is a unique closest A-world v to w. Since $B \lor C$ is true at v, either B or C must be true at v. So either B is true at all closest A-worlds to w or C is true at all closest A-worlds to v.

Exercise 8.15

'All dogs are barking': $\forall x(Dx \to Bx)$
'Some dogs are barking': $\exists x(Dx \land Bx)$
'Most dogs are barking' cannot be translated in terms of $\mathsf{M}x$. We need a binary

quantifier: $\mathsf{M}\,x(Bx/Dx)$

Exercise 8.16

On this proposal, bare indicative conditionals like (8) are material conditionals. If p is true and q is false then there is an accessible p-world at which q is false, and so q is not true at all accessible worlds at which p is true. In all other cases, q *is* true at all accessible worlds at which p is true.

Exercise 8.17

Conditional Excluded Middle is valid iff there is never more than one closest/accessible A-world. On that assumption, 'some closest/accessible A-world is a B-world' entails 'all closest/accessible A-worlds are B-worlds'. But (10) does not entail 'If I had played the lottery, I would have won'.